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AEROSOL TRANSPORT IN A NATURALLY-CONVECTED BOUNDARY LAYER*

J. M. HALES, L. C. SCHWENDIMAN and T. W. HORST

Atmospheric Analysis Section, Atmospheric Sciences Department, Battelle Memorial Institute,
Pacific Northwest Laboratories, Richland, Washington, U.S.A.

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Abstract—This paper is addressed to the analysis of aerosol deposition to an isothermal vertical surface through naturally-convected boundary layers. Both laminar and turbulent boundary layers, and condensing and noncondensing systems are considered. The general approach is to utilize a heat-transfer analogy to predict deposition in limiting cases of zero and predominant aerosol diffusion. These limiting results are then combined in an attempt to predict nonlimiting behavior.

Comparison with computed results from a more refined boundary-layer analysis of the steam–air system shows reasonable agreement for most situations involving laminar flows. No such comparison is possible for the turbulent regime, owing to the absence of related work in the field.

The equations presented herein are suggested for use in engineering estimates and as a means for correlating future related results of an experimental nature.

NOMENCLATURE

a,	constant defined by equation (40);
b,	constant defined by equation (41);
C,	constant defined by equation (25) [$\text{ft}^{-\frac{3}{4}}$];
C_B,	dimensionless correction factor in equation (9);
C_p,	specific heat of gas mixture [Btu/lbm °F];
C_{pg},	specific heat of gas [Btu/lbm °F];
C_{pw},	specific heat of condensable vapor [Btu/lbm °F];
D_p,	aerosol diffusion coefficient [ft^2/h];
D_{AB},	gas–vapor diffusion coefficient [ft^2/h];
F,	dimensionless variable defined by equation (23);
g,	gravitational acceleration [ft/h^2];
Gr_h,	thermal Grashoff number, defined by equation (73);
Gr_m,	mass Grashoff number defined by equation (74);
h,	local heat-transfer coefficient [$\text{Btu}/\text{ft}^2\text{h}°\text{F}$];
k,	thermal conductivity [Btu/fth °F];
k_x,	local mass transfer coefficient [$\text{lb-moles}/\text{hft}^2$];
$\dot{m}_h$,	local interfacial flux of energy away from wall [$\text{Btu}/\text{ft}^2\text{h}$];
$\dot{m}'_h$,	local interface flux of energy through gaseous boundary layer by conduction [$\text{Btu}/\text{ft}^2\ \text{h}$];
$\dot{m}_p$,	local interfacial flux of aerosol particles away from wall [$\text{particles}/\text{ft}^2\ \text{h}$];
$\dot{m}_w$,	local interfacial flux of vapor away from wall [$\text{lbm}/\text{ft}^2\text{h}$];
M,	average molecular weight, M_A = molecular weight of gas, M_B = molecular weight of vapor;
N,	aerosol particle concentration [$\text{particles}/\text{ft}^3$];
N_B,	dimensionless local Brownian transport number;
N_d,	dimensionless local diffusiophoretic transport number;
N_0,	dimensionless local net transport number;

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N_t,	dimensionless local thermophoretic transport number;
Nu,	local Nusselt number;
P,	total pressure [lbf/ft^2];
Pr,	Prandtl number;
q,	dimensionless constant defined by equation (38);
q_t,	local interfacial flux of energy away from wall [Btu/ft^{2}h] $q_t = \dot{m}_h$;
$Re_{\max}$,	Reynolds number, $v_{\max}\, x/\nu_i$;
Sc,	Schmidt number, ν/D_{AB};
Sh,	local Sherwood number, $k_x x/\rho D_{AB}$;
T,	temperature [°F];
T_n,	weighted transport ratio given by equation (72);
u,	down-plate mass-average velocity [ft/h];
v,	outward, normal mass-average velocity [ft/h];
W,	mass fraction of condensable vapor;
x, y,	distance variables defined in Fig. 1 [ft];
β,	constant defined by equation (37);
δ,	liquid-film thickness [ft];
Δ,	operator denoting the difference between bulk and interfacial conditions;
η,	dimensionless transformation variable defined by equation (24);
λ,	latent heat of vaporization of vapor [Btu/lbm];
μ,	dynamic viscosity [lbm/ft h];
ν,	kinematic viscosity, ft^2/hr;
ξ,	normalized mass fraction of vapor, $(W - W_i)/(W_\infty - W_i)$;
π,	density ratio, ρ_∞/ρ;
ρ,	density [lbm/ft^3];
Σ_1, Σ_2,	dimensionless variables defined by equations (34) and (35);
τ,	normalized temperature, $(T - T_i)/(T_\infty - T_i)$;
ϕ_c,	specific heat ratio, $C_p/C_{p\infty}$;
ϕ_{cvg},	specific heat difference, $(C_{pv} - C_{pg})/C_{p\infty}$;
ϕ_μ,	viscosity ratio, μ/μ_∞;
ψ,	steam function defined by equations (22a, b);
Υ,	normalized particle concentration, $(N - N_i)/(N_\infty - N_i)$.

Subscripts

A,	noncondensable gas;
B,	Brownian, also condensable vapor;
c,	diffusiophoretic correction;
d,	diffusiophoresis;
dep,	deposition;
g,	gas;
i,	interface;
L,	liquid;
p,	aerosol particle;
v,	vapor.

INTRODUCTION

PHORETIC transport of suspended particulate matter has been the subject of considerable investigation during recent years. Theoretical expressions, which have been developed to predict thermophoretic and diffusiophoretic transport rates, have been shown to agree with experimental data with moderate precision. Comparatively little work, however, has been devoted to the theoretical analysis of phoretic effects in conjunction with some of the more complex situations of engineering interest. The present study involves an analysis of one such situation—phoretic transport through a naturally-convected boundary layer adjacent to a vertical wall.

The physical situation under consideration here is one with considerable practical application, particularly in the field of nuclear-reactor containment-vessel design and accident analysis. Although the results presented in this paper relate directly to situations involving naturally-convected boundary layers and vertical walls only, the techniques of analysis employed herein can be extended to many other situations of engineering interest.

This paper begins with a discussion of the formulation and solution of laminar boundary-layer equations which characterize aerosol transport phenomena under the previously stated conditions. These solutions, which pertain specifically to steam–air mixtures, are then employed to develop a generalized correlation of aerosol deposition rates in the laminar regime. Finally, this approach is extended to develop a method of predicting deposition under conditions wherein turbulent boundary layers exist.

GOVERNING EQUATIONS

Figure 1 gives the basis for formulation of the boundary-layer equations of conservation to be utilized in this section. A bulk mixture of condensable vapor, gas and aerosol is exposed to the condensation surface, which is maintained at the temperature, T_w. Vapor from the bulk mixture migrates to the wall, forming a liquid film which runs off under the force of gravity. The convention will be used that energy, vapor, and aerosol fluxes away from the wall are positive entities.

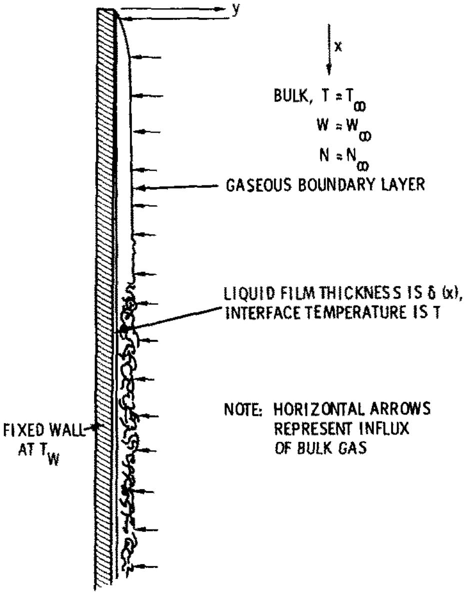

FIG. 1. Schematic of boundary layer formation adjacent to a vertical wall.

Presence of the liquid film in addition to the gaseous boundary layer causes heat transfer to occur as a two-step process. By making some reasonable assumptions one can reduce the complexity of the mathematics associated with this process, allowing solutions for each step to be obtained individually. Once formulated, these solutions can be matched at the interface, giving a composite description of the total system.

Heat transfer through a liquid film formed upon condensation of a pure vapor has been treated in the well-known derivation by Nusselt [1]. Nusselt's derivation may be extended to situations wherein a noncondensable gas is present, provided that the gas–liquid interface temperature, T_i, is independent of position. Assumption of constant T_i is allowable whenever the temperature drop across the liquid film is small compared to that across the gaseous boundary-layer—a condition that is met for almost all situations of interest in the context of the present analysis.

Description of the gaseous boundary layer is accomplished using the appropriate forms of the equations for conservation of mass, momentum, and energy. Assuming zero viscous dissipation, zero longitudinal diffusion, and negligible coupled transport effects such as thermal diffusion and diffusion thermo, Minkowycz and Sparrow [2] have listed these equations for the situation wherein aerosol is absent. These equations are valid for aerosol-containing systems, provided the presence of aerosol does not affect significantly the bulk properties of the gas. The equations are listed as follows:

Total Mass:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0, \qquad (1)$$

Mass of Vapor:

$$\rho u \frac{\partial W}{\partial x} + \rho v \frac{\partial W}{\partial y} = \frac{\partial}{\partial y}\left(\rho D_{AB} \frac{\partial W}{\partial y}\right), \qquad (2)$$

Momentum:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = g(\rho - \rho_\infty) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right), \qquad (3)$$

and Energy:

$$\rho C_p u \frac{\partial T}{\partial x} + \rho C_p v \frac{\partial T}{\partial y} + (C_{pg} - C_{pv}) \rho D_{AB} \times \frac{\partial W}{\partial y} \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right). \tag{4}$$

The nomenclature used in equations (1)–(4) is consistent with that found in most published treatments of this subject, and a complete definition of the symbols is given in the Nomenclature. However, it should be noted that here W denotes the mass fraction of vapor. This contrasts to the W of Minkowycz and Sparrow, which denotes the mass fraction of noncondensable gas.

For a complete description of aerosol-containing systems an additional equation is required. This accounts for conservation of the aerosol species, and may be written as follows:

$$\frac{\partial N v_p}{\partial y} + \frac{\partial N u_p}{\partial x} = 0, \tag{5}$$

where N is the number concentration of particles, and v_p and u_p are the mean particle velocities in the y and x directions, respectively. Equation (5) is valid for any aerosol under steady-state conditions, provided that no generation or decay occurs.

In view of the physical situation of interest, it is justifiable to let the x-component of the particle velocity equal the mass-average velocity in that direction, i.e.

$$u_p = u. \tag{6}$$

Particles moving in the y direction also are influenced by the bulk velocity. Here, however, correction terms arising from thermophoresis, counter-diffusion of gas and vapor, and particle diffusion may become important. Assuming an additive relationship, the net effect of these contributions may be represented as

$$v_p = v + v_c + v_t - \frac{D_p}{N} \frac{\partial N}{\partial y}, \tag{7}$$

where D_p is the aerosol diffusion coefficient, giving the aerosol diffusion flux in terms of a mass-fixed frame of reference.

The diffusiophoretic correction term, v_c, depends on the aerosol-particle size and its relation to the mean-free-path of the gas molecules. For conditions wherein the mean-free-path is large compared to particle size, (free-molecule regime), Waldmann [4] has developed the following equation:*

$$v_c = -\frac{D_{AB}}{W + \dfrac{1}{\sqrt{(M_A/M_B)} - 1}} \frac{\partial W}{\partial y}. \tag{8}$$

Free-molecule conditions will be assumed throughout this study.

Brock [5] has formulated a corresponding expression for the thermophoretic contribution to particle velocity. This may be expressed as:

$$v_t = -\frac{C_B k}{5P} \frac{\partial T}{\partial y}, \tag{9}$$

where k is the thermal conductivity and P is total system pressure. C_B is a correction term which may be estimated from Brocks original paper. For free-molecule conditions this term is of the order of unity.†

With these substitutions equation (5) becomes:

$$\frac{\partial (Nv)}{\partial y} + \frac{\partial (Nu)}{\partial x} - \frac{\partial}{\partial y}\left[\frac{N D_{AB}}{W + \dfrac{1}{\sqrt{(M_A/M_B)} - 1}} \frac{\partial W}{\partial y}\right] - \frac{\partial}{\partial y}\left[\frac{Nk}{5P} \frac{C_B \partial T}{\partial y}\right] = \frac{\partial}{\partial y}\left[D_p \frac{\partial N}{\partial y}\right], \tag{10}$$

completing the formulation of conservation equations for the gaseous boundary layer.

FORMULATION OF BOUNDARY CONDITIONS

The liquid-film description given by Nusselt can be matched conveniently to the foregoing

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* Equation (8) appears in a form somewhat different from that given by earlier authors. This is because of the necessity to express the correction in terms of the mass-average velocity, v, rather than the molar-average velocity which is used traditionally.

† C_B is assumed to be unity throughout this paper.

system of equations through boundary-conditions at the gas–liquid interface. Neglecting interfacial resistance, one may write immediately,

$$T = T_i \text{ at } y = \delta, \tag{11}$$

and

$$W = W_i \text{ (saturation value at } T_i\text{) at } y = \delta. \tag{12}$$

As a consequence of the absence of interfacial slip,

$$u_L = u_G \text{ at } y = \delta. \tag{13}$$

In terms of Nusselt's model this becomes,

$$u_G = \left[-\frac{q_t\,\delta x g}{\lambda\ \ \mu_L}\right]^{\frac{1}{2}} \text{ at } y = \delta, \tag{14}$$

where q_t is the total heat flux from the wall and λ is the latent heat of vaporization.

In addition, conservation of noncondensable gas and energy at the gas–liquid interface gives

$$\rho u \frac{\mathrm{d}\delta}{\mathrm{d}x} - \rho v = \frac{\rho D_{AB}}{1 - W_i}\frac{\partial W}{\partial y} \text{ at } y = \delta, \tag{15}$$

and

$$\frac{-\rho D_{AB}}{1 - W_i}\frac{\partial W}{\partial y}\lambda - k\frac{\partial T}{\partial y} = q_t \text{ at } y = \delta, \tag{16}$$

respectively.

Finally, a description of aerosol concentration at the interface is required. This is given simply by defining an interface concentration,

$$N = N_i \text{ at } y = \delta. \tag{17}$$

Boundary conditions at infinity depend on the assumption of a well-mixed bulk. These are largely self-explanatory, and are given as follows:

$$T = T_\infty \text{ at } y = \infty, \tag{18}$$

$$W = W_\infty \text{ at } y = \infty, \tag{19}$$

$$N = N_\infty \text{ at } y = \infty, \tag{20}$$

$$u = 0 \text{ at } y = \infty, \tag{21}$$

completing the description of boundary conditions for the gaseous boundary-layer model.

SIMILARITY TRANSFORMATION

Minkowycz and Sparrow [2] have introduced a similarity transformation for the gas–vapor boundary-layer problem described in the preceding sections. In applying the transformation, these authors utilized the conventional Von Mises transformation, introducing a stream function, Ψ, defined so as to satisfy the total continuity equation

$$u = \pi\frac{\partial \Psi}{\partial y}; \quad v = -\pi\frac{\partial \Psi}{\partial x}. \tag{22a, b}$$

Here π is the density ratio, ρ_∞/ρ.

These authors proceeded to define a new dependent variable, F, in terms of the stream function, having the quality of depending solely on the transformation variable, η:

$$F(\eta) = \frac{\Psi}{4\nu_\infty x^{\frac{3}{4}} C}, \tag{23}$$

$$\eta = Cx^{-\frac{1}{4}}\int_\delta^y \frac{\mathrm{d}y}{\phi_\mu}, \tag{24}$$

where

$$C = \left(\frac{q}{4\nu_\infty^2}\right)^{\frac{1}{4}} \tag{25}$$

and

$$\phi_\mu = \frac{\mu}{\mu_\infty}. \tag{26}$$

In addition, one can express vapor concentration, temperature, and aerosol population in terms of normalized variables,

$$\xi = \frac{W - W_i}{W_\infty - W_i}, \tag{27}$$

and

$$\tau = \frac{T - T_i}{T_\infty - T_i}, \tag{28}$$

$$\Upsilon = \frac{N - N_i}{N_\infty - N_i}. \tag{29}$$

Minkowycz and Sparrow's transformation

can be applied to equations (2), (3) and (4) to obtain:

$$\xi'' = -3Sc\xi' F, \tag{30}$$

$$\left(\frac{\tau'}{\phi_\mu}\right)' = -3Pr_\infty\left(\phi_c F + \frac{(\Delta W)\Phi_{cvg}}{3Sc}\xi'\right)\tau', \tag{31}$$

$$\left(\frac{\pi F'}{\phi_\mu}\right)'' + 3F\left(\frac{\pi F'}{\phi_\mu}\right)' - \frac{2\pi F'^2}{\phi_\mu} = \phi_\mu\left(1 - \frac{1}{\pi}\right). \tag{32}$$

Here the dependent variables are somewhat different from some of those chosen by Minkowycz and Sparrow. Also, the assumptions of constant k and Sc have entered the derivation.*

Equation (10) also can be expressed solely in terms of η, the result being given by:

$$\beta\frac{Sc}{\nu_\infty}\Upsilon'' = \Sigma_1\Upsilon' + \Sigma_2\left(\Upsilon + \frac{N_i}{N_\infty - N_i}\right), \tag{33}$$

where

$$\Sigma_1 = -\phi_\mu^2\left[A_0\pi + \frac{k\Delta TSc}{5P\nu_\infty}\left(\frac{\tau'}{\phi_\mu}\right)\right] + \frac{2\beta Sc\phi'_\mu}{\nu_\infty\phi_\mu}, \tag{34}$$

$$\Sigma_2 = -\phi_\mu^2\left[A_0(\pi' - \pi\Delta Wq\xi') + \frac{k\Delta TSc}{5P\nu_\infty} \times \left(\frac{\tau'}{\phi_\mu}\right)'\right], \tag{35}$$

$$A_0 = 3ScF + (\Delta W)q\xi', \tag{36}$$

$$\beta = D_p\phi_\mu, \tag{37}$$

and

$$q = \frac{1}{W + \dfrac{1}{\sqrt{(M_A/M_B)} - 1}}. \tag{38}$$

Formulation of equation (33) incorporates the assumption of constant β, which, in view of Stokes–Einstein behavior, should not introduce appreciable error when temperature limits are small.

To complete a description of the system of transformed equations, an equation of state is required. Assuming ideal-gas behavior, one can satisfy this requirement by writing

$$\pi = \frac{(\Delta T)\tau + T_i}{T_\infty}\left[a + b(\Delta W\xi + W_i)\right], \tag{39}$$

where

$$a = \frac{M_B}{M_B - W(M_B - M_A)}, \tag{40}$$

and

$$b = \frac{M_A - M_B}{M_B - W(M_B - M_A)} \tag{41}$$

In equations (30)–(33) the primes denote differentiation with respect to the variable η; hence the set of partial differential equations has been converted, through the similarity transformation, to an equivalent set of ordinary differential equations.

At this point the ultimate success of the similarity transformation depends on its ability to express the boundary conditions solely in terms of η. From equations (11), (12) and (17)–(21) it is obvious that

$$\tau = 0 \text{ at } \eta = 0, \tag{42}$$

$$\xi = 0 \text{ at } \eta = 0, \tag{43}$$

$$\Upsilon = 0 \text{ at } \eta = 0, \tag{44}$$

$$\tau = 1 \text{ at } \eta = \infty, \tag{45}$$

$$\xi = 1 \text{ at } \eta = \infty, \tag{46}$$

$$\Upsilon = 1 \text{ at } \eta = \infty, \text{ and} \tag{47}$$

$$F' = 0 \text{ at } \eta = \infty. \tag{48}$$

In addition, transformation of equation (15) provides the condition:

$$F = \frac{(\Delta W)\xi'}{3Sc(1 - W_i)} \text{ at } \eta = 0. \tag{49}$$

Finally, the no-slip condition (14) can be combined with (23) to give:

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* This contrasts with the work of Minkowycz and Sparrow, whose equations allowed for variable k and Sc.

$$F' = \left(-\frac{q_t\delta\phi_\mu^2}{4\lambda\mu_L\pi^2}\right)^{\frac{1}{2}} \text{ at } \eta = 0. \tag{50}$$

From Nusselt's derivation,

$$\delta = \left(-\frac{4q_t v_L^2 x}{\lambda\mu_L g}\right)^{\frac{1}{3}}. \tag{51}$$

Hence,

$$F' = \frac{\phi_\mu}{\pi}\left(-\frac{q_t}{\lambda\mu_L}\right)^{\frac{2}{3}}\left(\frac{v_L}{4}\right)^{\frac{1}{3}}\left(\frac{x}{g}\right)^{\frac{1}{6}} \text{ at } \eta = 0. \tag{52}$$

Upon transforming (16) and substituting into (52), one obtains

$$F' = r(s\tau' + t\xi')^{\frac{2}{3}} \text{ at } \eta = 0, \tag{53}$$

where

$$r = \frac{\phi_\mu}{2\pi}\left(\frac{v_L}{v_\infty}\right)^{\frac{1}{3}}(\lambda\mu_L)^{-\frac{2}{3}}, \tag{54}$$

$$s = k(\Delta T), \tag{55}$$

and

$$t = \frac{\mu\lambda(\Delta W)}{Sc(1 - W_i)}, \tag{56}$$

successfully transforming this final boundary condition and completing the statement of the transformed problem.

WALL-FLUX EQUATIONS

Because of their usefulness for relating solutions of the boundary-layer problem to macroscopic system behavior, the wall-flux equations are of primary importance to this investigation. Formulated by performing material and energy balances at the gas–liquid interface, these equations may be written:

Steam

$$\dot{m}_w = -\frac{C}{x^{\frac{1}{4}}}\left[\frac{\mu_\infty\xi\Delta W}{Sc(1 - W)}\right]_i \tag{57}$$

Energy

$$\dot{n}_h = -\frac{C}{x^{\frac{1}{4}}}\left[\frac{\lambda\mu_\infty(\Delta W)}{Sc(1 - W)}\xi' + \frac{C_p\mu_\infty(\Delta T)\tau'}{Pr}\right]_i \tag{58}$$

Aerosol

$$\dot{m}_p = -\frac{C}{x^{\frac{1}{4}}}\left[\left(3\pi v_\infty F + \frac{\pi q v_\infty \Delta W}{Sc}\xi' + \frac{C_B C_p \mu_\infty \Delta T}{5P\,Pr}\tau'\right)N + \frac{\beta N'}{\phi_\mu^2}\right]_i. \tag{59}$$

SOLUTION OF BOUNDARY LAYER PROBLEM

This problem was solved [3] for the steam–air system under several conditions of interest (c.f. Table 1) using an iterative technique on a hybrid computer, composed of a Beckman 2133 analog computer interfaced with a D.E.C. PDP-7 digital unit. Portions of the results are illustrated in Tables 2 and 3, which show computed fluxes at the gas–liquid interface.

These equations are expected to be quite valid for physical situations wherein truly laminar conditions exist and wherein the aerosol does not violate the stipulations regarding growth or decay set forth previously. From estimates of machine error and from digital checks of some of the solutions, the computation error is expected to be of the order of one per cent, based upon maximum values of the computed variables. Both the validity of the equations and the accuracy of this computation have been discussed in detail in a previous report [3].

Table 1. Physical conditions for computed results

Run*	$T_\infty - T_i$ (°R)	P_{steam} (atm)	T_∞ (°R)
1–1	1	1	671·7
1–4	4	1	671·7
1–16	16	1	671·7

Air pressure in atmospheres is equal to (T_∞, °R)/539·7 for all cases, corresponding to base conditions of one atmosphere air at 80°F.

* First term in run number designates bulk steam pressure in atmospheres. Second term denotes $T_\infty - T_i$, °F.

PREDICTIVE CORRELATION FOR DEPOSITION UNDER LAMINAR CONDITIONS

The aerosol deposition fluxes listed in Table 3

Table 2. Computed fluxes of energy and steam at liquid–vapor interface

Run	ξ_i'	τ_i'	$-\dot{m}_w x^{\frac{1}{4}}$ (lbm/ft$^{\frac{7}{4}}$ h)	$-\dot{m}_h x^{\frac{1}{4}}$ (Btu/ft$^{\frac{7}{4}}$ h)
1–1	0·127	0·149	0·0178	17·8
1–4	0·182	0·218	0·0951	95·1
1–16	0·267	0·338	0·439	445·0

pertain to steam–air systems under specific conditions. These results would be more meaningful if they could be extended to provide a means for calculating deposition rates under all conditions wherein naturally-convected laminar boundary layers exist adjacent to vertical walls. Such a correlation, which may be employed for noncondensing systems as well as those in which condensation occurs, is developed in the present section.

The procedure employed here is to visualize two limiting cases of deposition behavior, corresponding to predominant phoretic effects and predominant Brownian transport. Deposition fluxes under each of these conditions are estimated on the basis of a heat-transfer analogy, and the results are then combined to provide estimates of nonlimiting behavior.

*Table 3. Computed aerosol deposition fluxes**

Run†	γ_i'	D_{B}‡ (ft^2/h)	$-\dot{m}_p x^{\frac{1}{4}}/N_\infty$ (ft$^{\frac{5}{4}}$ h)
1–1–0·0025	0·685	0·0193	2·12
1–1–0·005	1·24	0·00490	0·993
1–1–0·01	2·00	0·00127	0·412
1–1–0·05	18·9	0·0000651	0·201
1–1–0·01	55·7	0·0000217	0·197
1–4–0·0025	1·23	0·0192	3·79
1–4–0·005	2·88	0·00487	2·26
1–4–0·01	6·01	0·00126	1·21
1–4–0·05	103·8	0·0000648	1·08
1–4–0·1	311·0	0·0000216	1·08
1–16–0·0025	3·65	0·0187	10·22
1–16–0·005	11·77	0·00474	8·56
1–16–0·01	29·2	0·00123	5·47
1–16–0·05	56·0	0·0000631	5·37
1–16–0·1	1681·0	0·0000210	5·36

* Per unit bulk concentration, based on zero interfacial aerosol concentration.

† Last term in run number denotes particle diameter in microns.

‡ Calculated from (corrected) relationship for aerosol diffusion coefficient given by Davies [14].

Limiting cases can be evaluated with the aid of the deposition-flux equations (57)–(59), or they can be analyzed directly by considering the limiting deposition velocities.

For the limiting case of predominant phoretic effects the aerosol concentration adjacent to the interface is expected to be close to N_∞. Thus the diffusiophoretic and thermophoretic deposition rates are given, in accordance with equation (7) by

$$m_{p,\,B=0} = (v + v_c + v_t)N_\infty. \tag{60}$$

Combining with (8) and (9), and expressing in dimensionless form gives

$$N_d = \frac{v_d x}{\nu}\bigg|_{\text{interface}} = -\frac{Sh\Delta W}{Sc} \times \left[\frac{1}{1 - W} + \frac{1}{W + 1/(\sqrt{M_A/M_B} - 1)}\right]\bigg|_{\text{interface}}, \tag{61}$$

(zero Brownian motion)

$$N_t = \frac{v_t x}{\nu}\bigg|_{\text{interface}} = -\frac{C_B M Nu C_p \Delta T}{5PrRT}\bigg|_{\text{interface}} \tag{62}$$

(zero Brownian motion).

N_d and N_t, which are essentially Reynolds numbers for the individual mechanisms, shall be referred to as the "local diffusiophoretic and thermophoretic transport numbers," respectively. The Sherwood and Nusselt numbers, in accordance with the treatment by Bird *et al.* [6], are defined here by the relations

local interfacial mass flux of condensing vapor

$$= -\frac{ShD_{AB}\rho\Delta W}{x(1 - W)} \tag{63}$$

and

local interfacial conduction-heat flux

$$= -\frac{Nuk\Delta T}{x}. \tag{64}$$

Similarly, by defining a particle Sherwood number,

local interfacial number flux of particles

$$= -\frac{Sh_p D_p N_\infty}{x}, \qquad (65)$$

one can obtain a corresponding expression for the local particle transport number in the limit of predominant Brownian motion (zero phoresis);

$$N_B = \left.\frac{v_B x}{v}\right|_{\text{interface}} = -\left.\frac{Sh_p}{Sc_p}\right|_{\text{interface}}. \qquad (66)$$

(zero phoresis)

Here the Schmidt and Prandtl numbers, Sc, Sc_p and Pr are defined by v_i/D_{AB}, v_i/D_p and $C_p\mu/k$, respectively, where v_i denotes interfacial kinematic viscosity and C_p is the specific heat of the gas mixture. v_B may be regarded as a "deposition velocity" arising from Brownian effects. In addition one can define net local transport number in terms of total deposition velocity,

$$N_0 = \left.\frac{v_{\text{dep}} x}{v}\right|_{\text{interface}}, \qquad (67)$$

to account for the net deposition process. Intuitively one would expect N_0 to vary as a function of the limiting values, i.e.

$$N_0 = f(N_d, N_t, N_B), \qquad (68)$$

although the nature of the function f remains unresolved at this stage.

Aerosol deposition fluxes corresponding to the limiting situations of zero Brownian motion and zero phoresis can be determined from equations (61), (62) and (66) provided that Nu, Sh and Sh_p are known. Because of similarity of the processes of heat, vapor and aerosol transport, these entities can be estimated on the basis of analogy to heat transfer through laminar bounddary layers adjacent to vertical walls in non-condensing systems. Under such conditions the expression

$$Nu = 0{\cdot}388\, Gr_h^{\frac{1}{4}} Pr^{\frac{1}{4}}, \qquad (69)$$

has been shown to provide an adequate representation of local heat-transfer behavior [7]. Here Gr_h is the Grashoff number, which represents the ratio of buoyancy and viscous forces, [c.f. equation (73)].

Equation (69) may be compared with the form

$$Nu = aRe^{\alpha 1} Pr^{\alpha 2};\ a, \alpha_i \text{ constant}, \qquad (70)$$

which arises often during the analysis of forced-convection heat-transfer problems. Equations (69) and (70) are similar in that they both attempt to represent heat transfer as a function of diffusional and flow-field characteristics. There is a basis difference, however. In equation (70) the flow-field and the diffusivity are represented explicitly in terms of Re and Pr, respectively. For the case of equation (69), Pr represents the diffusive character of the system; but it is important also in determining flow behavior adjacent to the wall. For simultaneous heat and mass transport the analogy can be utilized considering individually the transport rates of energy and mass, but it is important to recognize that in either case the flow pattern is influenced by both. For this reason it seems expedient to attempt a separation of the characterizations of the flow field and diffusion processes into explicit terms prior to drawing any such analogy and applying it to equation (69).

Such a separation may be attempted by rewriting (69) in the form of (70). For this Re must be determined as a function of Pr and Gr_h. If simultaneous heat and mass transfer is involved, Re must be expressed in terms of Pr, Sc and Gr, where Gr now takes into account the effects of both temperature and composition gradients. If Re is defined in terms of the *maximum* velocity in the boundary layer, an analysis of the existing theoretical and experimental results [3, 7, 8] for pure and simultaneous heat and mass transfer shows that the Reynolds number can be expressed adequately by the form

$$Re_{\max} = \frac{v_{\max} x}{v_i}$$

$$= 0{\cdot}49(Gr_h + Gr_m)^{\frac{1}{2}} Tn^{-0\cdot38} \qquad (71)$$

where Tn is a weighted transport ratio given by

$$T_n = \frac{Gr_h Pr + Gr_m Sc}{Gr_h + Gr_m}. \tag{72}$$

Here, Gr_h and Gr_m are Grashoff numbers arising from temperature and concentration gradients, respectively.

$$Gr_h = \frac{x^3 g \Delta T}{v_i^2 T} \tag{73}$$

and

$$Gr_m = \frac{x^3 g \Delta W}{v_i^2 \left(\dfrac{1}{1 - M_A/M_B} - W\right)}. \tag{74}$$

From equations (69) and (71) one obtains

$$\begin{aligned} Nu &= 0{\cdot}555 \sqrt{Re_{\max}}\, Pr^{0{\cdot}44} \\ &= 0{\cdot}388(Gr_h + Gr_m)^{\frac{1}{4}}\, Tn^{-0{\cdot}19} Pr^{0{\cdot}44}. \end{aligned} \tag{75}$$

By analogy of mass transfer to heat transfer one may write

$$Sh = 0{\cdot}388(Gr_h + Gr_m)^{\frac{1}{4}}\, Tn^{-0{\cdot}19} Sc^{0{\cdot}44} \tag{76}$$

and

$$Sh_p = 0{\cdot}388(Gr_h + Gr_m)^{\frac{1}{4}}\, Tn^{-0{\cdot}19} Sc_p^{0{\cdot}44}, \tag{77}$$

providing the required expressions for Nu, Sh and Sh_p.

It should be noted that for gas–vapor systems wherein Sc and Pr are nearly equal (the air–steam system is a good example of this), the errors which arise from applying equation (69) to the analogy without separating flow and diffusive effects are small. Indeed, it has been shown [3] that mass-transfer rates for steam–air mixtures can be predicted from (69) to within a few per cent, demonstrating partially the validity of the overall method. For increasing differences between Sc and Pr, however, the errors become large, and (75) and (76), and (77) should be expected to provide a much more satisfactory representation of true physical behavior.

From the appropriate equations one now can estimate the limiting quantities, N_d, N_t and N_B for virtually any system, condensing or non-condensing, involving aerosol transport through laminar, naturally-convected boundary layers adjacent to vertical walls. This is contingent, of course, upon the condition that liquid film velocities, which are not considered in the analogy, are small. This estimation may be accomplished by utilizing equations (75)–(77) to compute Nu, Sh and Sh_p, and then employing groups in (61), (62) and (66) to determine the limiting aerosol transport numbers.

To complete and verify the generalized correlation the following must be answered.

1. How well do the two limiting expressions describe behavior under appropriate conditions?
2. How can the limiting expressions be combined to predict behavior under nonlimiting conditions?

Figure 2 provides an indication as to the first of these. Here computed deposition rates from the previous section are plotted as points along

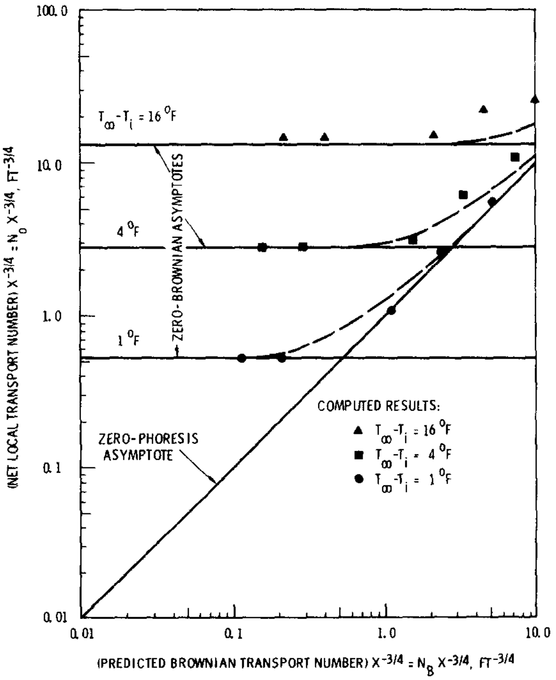

FIG. 2. Comparison of computed deposition rates with those predicted from analogy. Curves show film-theory approximation of non-limiting behavior. Calculations for bulk pressures of 2634 lb/ft² air and 2116 lb/ft² steam at 212°F.

with the (solid) lines describing limiting behavior determined on the basis of (61), (62) and (66). This good limiting agreement testifies to the power of the analogy, although it should be emphasized that this is not a comparison with actual physical behavior, but merely with results obtained through the more refined mathematical approach of the preceding section. The increasing deviation above the "predominant phoresis" asymptote at higher temperatures is thought to arise primarily because of liquid-film velocities, which become more important under these conditions.

Some insight pertaining to the second question can be obtained from visualization of the physical processes involved. Since both phoretic and diffusional processes act in the same direction under these circumstances, their net effect could be estimated by adding the limiting expressions, i.e.

$$N_0 = (N_d + N_t + N_B). \tag{78}$$

This may be considered a zero-order approximation to true behavior. Upon applying a film-theory analysis, a more satisfactory approximation is suggested·

$$N_0 \simeq \frac{N_{dt}}{\exp(-N_{dt}/N_B) - 1};$$
$$N_{dt} = N_d + N_t. \tag{79}$$

Equation (79) is represented by dashed lines in Fig. 2. Because of one-dimensionality, and because of other simplifications of the film-theory approach (79) cannot be expected to describe true behavior exactly; however, the agreement between this similarity approximation and the boundary-layer solutions is rather good, except for intermediate situations under high temperature-drop conditions. Such cases are of limited interest in most practical situations, since laminar boundary layers do not persist for large downwall distances under these circumstances.

At this point one might attempt introducing an empirical correction factor in (79) to force it to adhere more closely to the refined solutions. The correction factor,

$$C_f = 1 + a(1 - N_{dt}) \times \exp\left[-b\left|\left(\frac{N_B}{N_{dt}} - \frac{N_{dt}}{N_B}\right)\right|\right], \tag{80}$$

for instance, could be used to improve the agreement upon selecting appropriate values of the constants a and b. In view of the lack of experimental data and the relative scarcity of computed solutions, however, it is perhaps unmeaningful to attempt such an effort until more results, especially experimental ones, become available.

The success of this approximate analysis suggests that a number of other problems involving aerosol transport may benefit from similar treatment, although the advantages of this type of similarity approach are lessened considerably for situations where particle inertia is an important factor. Even under these circumstances, however, some advantage may be gained by performing a total analysis as an extension of a limiting situation where particle inertia is zero. In this regard the assumptions of the present analysis regarding zero gravitational and inertial effects associated with the particles, and negligible growth or decay of the aerosol should be emphasized. For physical systems where these assumptions are invalid, the present analysis should be expected to be less accurate, depending upon the degree of violation of the stipulated behaviour.

PREDICTION OF AEROSOL DEPOSITION UNDER TURBULENT CONDITIONS

The apparent success of the treatment by analogy of aerosol transport in laminar boundary layers suggests that a similar approach might be useful in examining the turbulent regime. This expectation, however, is tempered by a number of complications. Firstly, the problem of particle inertia, which has no analogous counterpart in heat transfer, will become more important whenever turbulent conditions prevail. Secondly, there is comparatively little experimental data concerning turbulent boundary layers that is useful for the purpose of developing an analogy-based relationship. Finally, there exists no means of testing the validity of any analogy in the turbulent regime, since neither boundary-layer solutions nor experimental deposition data are available for these conditions.

If one parallels the procedure of the previous section, the problem of determining the maximum Reynolds number is again encountered. Eckert and Jackson [9] have employed an integral boundary-layer analysis to obtain an expression for *Re* in terms of Gr_h and *Pr* for turbulent natural-convection heat transfer; however, their result agrees poorly with the few measured values that presently exist. Experimentally measured values [10, 11], on the other hand, are available for air only, and therefore do not give an indication of the influence of the Prandtl number on velocity.

Of these experimental studies the results of Cheesewright [10] are perhaps the most useful for present purposes, since they pertain to the largest values of *Gr*; and hence to situations well within the turbulent regime. From these results one may write

$$Re = 0{\cdot}295 Gr_h^{\frac{1}{2}} \text{ (for air).} \tag{81}$$

One should note here that the exponent $\frac{1}{2}$ is in agreement with the theory of Eckert and Jackson, although there is considerable deviation between the leading coefficients. The experimental results of Coutanceau [11] also indicate a square-root dependence on *Gr*, while providing marginal agreement with the numerical results of Cheesewright.

If, in accordance with (71), one assumes that the form

$$Re_{\max} = a(Gr_h + Gr_m)^{\frac{1}{2}}\, Tn^n, \tag{82}$$

(*a* and *n* unspecified) will provide a satisfactory representation of physical behavior, then the procedure of the previous section can be repeated. From turbulent-flow heat-transfer results one may write* [7]

$$Nu = 0{\cdot}0252 Gr_h^{\frac{2}{5}} Pr^{\frac{2}{5}} \tag{83}$$

which, upon application of (82) results in

$$\begin{aligned} Nu &= 0{\cdot}0252 a^{-\frac{4}{5}} Re_{\max}^{\frac{4}{5}} Pr^{(2-4n)/5} \\ &= 0{\cdot}0252(Gr_h + Gr_m)^{\frac{2}{5}}\, Tn^{(4n/5)} Pr^{(2-4n)/5}. \end{aligned} \tag{84}$$

Again, by similarity, one may write

$$Sh = 0{\cdot}0252(Gr_h + Gr_m)^{\frac{2}{5}}\, Tn^{(4n/5)} Sc^{(2-4n)/5}, \tag{85}$$

and

$$Sh_p = 0{\cdot}0252(Gr_h + Gr_m)^{\frac{2}{5}}\, Tn^{(4n/5)} Sc_p^{(2-4n)/5}. \tag{86}$$

Provided *n* is known, one may compute deposition rates from these equations by substituting into (61), (62), (66) and finally (79) in the same manner as for the laminar situation. As was mentioned previously, there is presently no reliable means of estimating *n*; the result of Eckert and Jackson, however, indicates that *n* should lie in the neighborhood of $-\frac{1}{3}$. Substituting this value into the preceding equations gives

$$Nu = 0{\cdot}0252(Gr_h + Gr_m)^{\frac{2}{5}}\, Tn^{-4/15} Pr^{\frac{2}{3}}, \tag{87}$$

$$Sh = 0{\cdot}0252(Gr_h + Gr_m)^{\frac{2}{5}}\, Tn^{-4/15} Sc^{\frac{2}{3}}, \tag{88}$$

and

$$Sh_p = 0{\cdot}0252(Gr_h + Gr_m)^{\frac{2}{5}}\, Tn^{-4/15} Sc_p^{\frac{2}{3}}, \tag{89}$$

which, employed in conjunction with (61), (62), (66) and (79), provide a means, albeit rather speculative, of estimating aerosol deposition under turbulent conditions. As should be expected, these equations tend to predict particle deposition rates greater than those that would be predicted by the laminar-flow equations for similar conditions. Deposition, in contrast to laminar-flow behavior, increases slowly with distance down the deposition surface. It is emphasized that equations (87)–(89) are tentative results, and should be modified accordingly when future information, particularly with regard to the value of *n*, becomes available.

Similar to the case for laminar boundary

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* Note that, in accordance with the treatment throughout this paper, *Nu* is the *local* Nusselt number.

layers, errors arising from the neglect of aerosol size changes may be significant depending upon specific physical conditions. Other authors [12, 13] have indicated that such changes may be appreciable under appropriate circumstances. The insensitivity of phoretic transport to particle size, however, indicates that related modifications in deposition behavior should be important only under conditions involving rather fine particles exhibiting a strong component of Brownian transport.

CONCLUSIONS

A generalized method has been described which enables the estimation of aerosol deposition to a vertical, isothermal surface through both turbulent and laminar naturally-convected boundary layers. The method is applicable to both noncondensing and condensing systems; liquid-film effects are a potential source of error in situations involving the latter, but such errors are expected to be of secondary importance in most cases of practical interest.

The equations presented herein have been found to compare with computed solutions to laminar boundary-layer equations reasonably well. No such comparison has been made for the turbulent regime, owing to the absence of a more refined analysis, either theoretical or experimental in nature. Despite these obvious difficulties, these equations are recommended for use whenever related engineering calculations are necessary. This recommendation is based partially upon the fact that there exists at present no other simple and reasonable means for approaching the problem; it is based also, however, upon the fact that the comparisons that are available tend to testify to the general validity of the procedure.

Obviously, experimental measurements of this type, not available at present, would be of high interest in this regard. Such results, when they become available, can be used directly to modify the equations of this paper to make them describe physical behavior more accurately. Conversely, the equations presented herein are suggested as a basis for correlating and analyzing future work of an experimental nature.

Finally, it should be noted that all derivations herein have pertained to local fluxes and homogeneous aerosols. Average fluxes, therefore, must be computed from the local values by simple integration over the vertical surface. Similarly, if the aerosol of interest is heterogeneous in nature, then results must be computed from the analysis of this paper for a number of aerosol sizes and distributed accordingly.

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TRANSPORT D'AEROSOL DANS UNE COUCHE LIMITE DE CONVECTION NATURELLE

Résumé—Cet article expose l'étude du dépôt d'aérosol sur une surface verticale isotherme à travers des couches limites de convection naturelle. On considère à la fois les couches limites laminaires et turbulentes et des systèmes condensables ou non. L'approche générale est l'utilisation d'une analogie thermique pour évaluer le dépôt dans des cas limites de diffusion d'aérosol nulle ou prédominante. Ces résultats limites sont alors combinés dans un essai de prédiction du comportement quelconque.

Une comparaison avec les résultats obtenus sur calculateur digital à partir d'une analyse plus fine de couche limite pour le système vapeur–air montre un accord raisonnable pour la plupart des situations impliquant des écoulements laminaires. Une telle comparaison n'est pas possible pour le régime turbulent en l'absence d'un travail correspondant dans le domaine.

Les équations présentées ici sont suggérées pour l'utilisation dans les projets industriels et comme étant des moyens de représentation de résultats expérimentaux ultérieurs.

AEROSOL-TRANSPORT IN GRENZSCHICHTEN BEI FREIER KONVEKTION

Zusammenfassung—Diese Arbeit befasst sich mit der Untersuchung von Aerosol-Ablagerungen an einer isothermen vertikalen Fläche mit einer Grenzschicht infolge freier Konvektion. Sowohl laminare und turbulente Grenzschichten, als auch kondensierende und nichtkondensierende Systeme wurden untersucht. Eine allgemeine Näherungsmethode ist die Verwendung einer Wärmeübertragungsanalogie, um Ablagerungen von sogenannten "Nullmengen" bei vorherrschender Aerosol-Diffusion vorherzusagen.

Diese Grenzwert-Lösungen werden dann kombiniert um eine Aussage für das Verhalten bei endlichen Mengen zu gewinnen.

Vergleiche mit numerischen Ergebnissen bei einer verfeinerten Grenzschicht-Analyse für den Fall eines Dampf-Luft-Systems zeigen annehmbare Übereinstimmung für die meisten Fälle mit laminarer Strömung. Für den turbulenten Bereich sind solche Vergleiche nicht möglich, was auf das Fehlen diesbezüglicher Arbeiten auf diesem Gebiet zurückzuführen ist. Die hier angegebenen Gleichungen werden zum Gebrauch bei ingenieurmässigen Abschätzungen und als Anhaltswerte für die Korrelation zukünftiger experimenteller Ergebnisse empfohlen.

ПЕРЕНОС АЭРОЗОЛИ В ПОГРАНИЧНОМ СЛОЕ С ЕСТЕСТВЕННОЙ КОНВЕКЦИЕЙ

Аннотация—Статья посвящена анализу отложения аэрозоли на изотермическую поверхность через пограничный слой естественной конвекции. Рассматриваются как ламинарные, так и турбулентные слои, а также конденсирующиеся и неконденсирующиеся системы. Общий подход состоит в использовании аналогии теплопереноса для расчета отложения аэрозоля в предельных случаях нулевой и преобладающей диффузии. Затем эти прдельные результаты используются при попытке расчета непредельного случая. Сравнение с результатами расчетов при более точном анализе пограничного слоя системы водяной пар- воздух показывает приемлемое соответствие для большинства случаев ламинарных течений. Такое сравнение невозможно для турбулентного режима из-за отсутствия соответствующих исследований в этой области. Предложены уравнения для инженерных оценок и для интерпретации будущих экспериментальных данных.